Sustained oscillations in simple epidemic models

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Abstract

Cycles are a very striking behaviour of prey-predator systems also seen in a variety of other host-enemy systems - a case in point is the well documented patterns of recurrent epidemics of measles and other childhood diseases in the pre-vaccination era. Recently, an exceedingly simple and general mechanism of resonant amplification of demographic stochasticity has been proposed to describe the cycling behaviour of prey-predator systems [3]. The resonant mechanism is generic for a class of stochastic systems that includes the majority of the classical models of diseases that confer either lifelong or temporary total immunity [2], and goes a long way in describing realistic patterns of the recurrent epidemics of measles and pertussis, in the absence of external forcing terms [1].

Here we consider the simplest epidemiological unforced model, the Susceptible-Infective-Recovered-Susceptible (SIRS) model, where immunity is lost after a given time, and suggest that it may exhibit two types of cycling behaviour, i.e. in a certain range of parameters the model exhibits resonant amplification of stochastic fluctuations as in [3] and in a narrow range of parameters, that is relevant in the epidemiological context, the limit cycles persist for infinite populations. We analyse the dynamical transition (or bifurcation) that separates the stochastic resonance regime from the oscillatory phase using an uncorrelated pair approximation. Analogous behaviour is found in the related demographic Susceptible-Infective-Recovered (dSIR) model [4], where the pool of susceptibles is replenished through births.

We provide further evidence of the phase diagram based on the analytical pair approximation with results for the power spectra calculated through Monte Carlo simulations of the individually based model interacting through a regular random graph (RRG) with 4 links per node, for large population sizes N.The latter exhibits distinctive behaviour in a region of parameter space, characterized by a low rate of replenishment of susceptibles, consistent with the presence of stable cycles in the limit of infinite populations. We also relate our findings for the presence of an oscillatory phase with those previously published for the same model on a square lattice [5].

Finally, we discuss results for various prey-predator models where cycling behaviour, in the limit of infinite systems, may also be observed [3, 6].

References

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